$$1 : \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$m : \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$2 : \begin{pmatrix} \cos \frac{2\pi}{2} & -\sin \frac{2\pi}{2} & & \\ \sin \frac{2\pi}{2} & \cos \frac{2\pi}{2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$3 : \begin{pmatrix} \cos \frac{2\pi}{3} & -\sin \frac{2\pi}{3} & & \\ \sin \frac{2\pi}{3} & \cos \frac{2\pi}{3} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$4 : \begin{pmatrix} \cos \frac{2\pi}{4} & -\sin \frac{2\pi}{4} & & \\ \sin \frac{2\pi}{4} & \cos \frac{2\pi}{4} & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$6 : \begin{pmatrix} \cos \frac{2\pi}{6} & -\sin \frac{2\pi}{6} & & \\ \sin \frac{2\pi}{6} & \cos \frac{2\pi}{6} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\bar{1} : \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

$$\bar{3} : -\begin{pmatrix} \cos \frac{2\pi}{3} & -\sin \frac{2\pi}{3} & & \\ \sin \frac{2\pi}{3} & \cos \frac{2\pi}{3} & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} \cos \frac{-2\pi}{6} & -\sin \frac{-2\pi}{6} & & \\ \sin \frac{-2\pi}{6} & \cos \frac{-2\pi}{6} & & \\ & & -1 & \\ & & & 1 \end{pmatrix} : 6(m)$$

$$\bar{4} : -\begin{pmatrix} \cos \frac{2\pi}{4} & -\sin \frac{2\pi}{4} & & \\ \sin \frac{2\pi}{4} & \cos \frac{2\pi}{4} & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} \cos \frac{-2\pi}{4} & -\sin \frac{-2\pi}{4} & & \\ \sin \frac{-2\pi}{4} & \cos \frac{-2\pi}{4} & & \\ & & -1 & \\ & & & 1 \end{pmatrix} : 4(m)$$

$$\bar{6} : -\begin{pmatrix} \cos \frac{2\pi}{6} & -\sin \frac{2\pi}{6} & & \\ \sin \frac{2\pi}{6} & \cos \frac{2\pi}{6} & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} \cos \frac{-2\pi}{3} & -\sin \frac{-2\pi}{3} & & \\ \sin \frac{-2\pi}{3} & \cos \frac{-2\pi}{3} & & \\ & & -1 & \\ & & & 1 \end{pmatrix} : 3(m)$$

$$\bar{1}_4 : \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$3(3) : \begin{pmatrix} \cos \frac{2\pi}{3} & -\sin \frac{2\pi}{3} & & \\ \sin \frac{2\pi}{3} & \cos \frac{2\pi}{3} & & \\ & & \cos \frac{2\pi}{3} & -\sin \frac{2\pi}{3} \\ & & \sin \frac{2\pi}{3} & \cos \frac{2\pi}{3} \end{pmatrix}$$

$$3(2): \begin{pmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & & \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & & \\ & & \cos\frac{2\pi}{2} & -\sin\frac{2\pi}{2} \\ & & \sin\frac{2\pi}{2} & \cos\frac{2\pi}{2} \end{pmatrix} = \begin{pmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & & \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$4(4): \begin{pmatrix} \cos\frac{2\pi}{4} & -\sin\frac{2\pi}{4} & & \\ \sin\frac{2\pi}{4} & \cos\frac{2\pi}{4} & & \\ & & \cos\frac{2\pi}{4} & -\sin\frac{2\pi}{4} \\ & & \sin\frac{2\pi}{4} & \cos\frac{2\pi}{4} \end{pmatrix} = \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & -1 \\ & & 1 & 0 \end{pmatrix}$$

$$4(3): \begin{pmatrix} \cos\frac{2\pi}{4} & -\sin\frac{2\pi}{4} & & \\ \sin\frac{2\pi}{4} & \cos\frac{2\pi}{4} & & \\ & & \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ & & \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ & & \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{pmatrix}$$

$$4(2): \begin{pmatrix} \cos\frac{2\pi}{4} & -\sin\frac{2\pi}{4} & & \\ \sin\frac{2\pi}{4} & \cos\frac{2\pi}{4} & & \\ & & \cos\frac{2\pi}{2} & -\sin\frac{2\pi}{2} \\ & & \sin\frac{2\pi}{2} & \cos\frac{2\pi}{2} \end{pmatrix} = \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$6(6): \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} \\ & & \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} \end{pmatrix}$$

$$6(4): \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & \cos\frac{2\pi}{4} & -\sin\frac{2\pi}{4} \\ & & \sin\frac{2\pi}{4} & \cos\frac{2\pi}{4} \end{pmatrix} = \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & 0 & -1 \\ & & 1 & 0 \end{pmatrix}$$

$$6(3): \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ & & \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{pmatrix}$$

$$6(2): \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & \cos\frac{2\pi}{2} & -\sin\frac{2\pi}{2} \\ & & \sin\frac{2\pi}{2} & \cos\frac{2\pi}{2} \end{pmatrix} = \begin{pmatrix} \cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6} & & \\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6} & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$[5] = 55^2: \begin{pmatrix} \cos\frac{2\pi}{5} & -\sin\frac{2\pi}{5} & & \\ \sin\frac{2\pi}{5} & \cos\frac{2\pi}{5} & & \\ & & \cos\frac{4\pi}{5} & -\sin\frac{4\pi}{5} \\ & & \sin\frac{4\pi}{5} & \cos\frac{4\pi}{5} \end{pmatrix}$$

$$[8] = 88^3: \begin{pmatrix} \cos\frac{2\pi}{8} & -\sin\frac{2\pi}{8} & & \\ \sin\frac{2\pi}{8} & \cos\frac{2\pi}{8} & & \\ & & \cos\frac{6\pi}{8} & -\sin\frac{6\pi}{8} \\ & & \sin\frac{6\pi}{8} & \cos\frac{6\pi}{8} \end{pmatrix}$$

$$[10] = 1010^3: \begin{pmatrix} \cos\frac{2\pi}{10} & -\sin\frac{2\pi}{10} & & \\ \sin\frac{2\pi}{10} & \cos\frac{2\pi}{10} & & \\ & & \cos\frac{6\pi}{10} & -\sin\frac{6\pi}{10} \\ & & \sin\frac{6\pi}{10} & \cos\frac{6\pi}{10} \end{pmatrix}$$

$$[12] = 1212^5: \begin{pmatrix} \cos\frac{2\pi}{12} & -\sin\frac{2\pi}{12} & & \\ \sin\frac{2\pi}{12} & \cos\frac{2\pi}{12} & & \\ & & \cos\frac{10\pi}{12} & -\sin\frac{10\pi}{12} \\ & & \sin\frac{10\pi}{12} & \cos\frac{10\pi}{12} \end{pmatrix}$$

symbol	trace	determinant	second invariant
1	4	1	6
m	2	-1	0
2	0	1	-2
3	1	1	0
4	2	1	2
6	3	1	4
$\bar{1}$	-2	-1	0
$\bar{3}$	1	-1	0
$\bar{4}$	0	-1	0
$\bar{6}$	-1	-1	0
$\bar{1}_4$	-4	1	6
3(3)	-2	1	3
3(2)	-3	1	4
4(4)	0	1	2
4(3)	-1	1	2
4(2)	-2	1	2
6(6)	2	1	3
6(4)	1	1	2
6(3)	0	1	1
6(2)	-1	1	0
[5]	-1	1	1
[8]	0	1	0
[10]	1	1	1
[12]	0	1	-1

second invariant

$$(s_{11}s_{22} - s_{12}s_{21}) + (s_{11}s_{33} - s_{13}s_{31}) + (s_{11}s_{44} - s_{14}s_{41})$$
$$+(s_{22}s_{33} - s_{23}s_{32}) + (s_{22}s_{44} - s_{24}s_{42}) + (s_{33}s_{44} - s_{34}s_{43})$$